Marking Scheme
Strictly Confidential
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Senior Secondary School Examination 2026
MATHEMATICS (041) [PAPER CODE -65(B)]

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its leakage to the public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in Newspaper/Website, etc. may invite action under various rules of the Board and BNS."
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. The Marking Scheme should be strictly adhered to and religiously followed. However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-XII, while evaluating the competency-based questions, please try to understand the given answer and even if reply is not from a marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.
4	The Marking Scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS 'X' be marked. Evaluators will not put right (✓) while evaluating which gives the impression that the answer is correct, and no marks are awarded. This is the most common mistake which evaluators are committing.
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	If a student has attempted an extra question, answer to the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".
10	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.

11	**A full scale of marks (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.**
12	**Every examiner must necessarily do evaluation work for full working hours, i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.**
13	**Ensure that you do not make the following common types of errors committed by the Examiner in the past: -** ● **Leaving answer or part thereof unassessed in an answer book.** ● **Giving more marks for an answer than assigned to it.** ● **Wrong totaling of marks awarded on an answer.** ● **Wrong transfer of marks from the inside pages of the answer book to the title page.** ● **Wrong question wise totaling on the title page.** ● **Wrong totaling of marks of the two columns on the title page.** ● **Wrong grand total.** ● **Marks in words and figures not tallying/not same.** ● **Wrong transfer of marks from the answer book to online award list.** ● **Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)** ● **Half or a part of the answer marked correct and the rest as wrong, but no marks awarded.**
14	**While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.**
15	**Any unassessed portion, non-carrying over of marks to the title page, or total error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.**
16	**The Examiners should acquaint themselves with the guidelines given in the "Guidelines for Spot Evaluation" before starting the actual evaluation.**
17	**Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.**
18	**The candidates are entitled to obtain a photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.**

MARKING SCHEME
MATHEMATICS (Subject Code–041)
(PAPER CODE: 65(B))

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	STEPS	MARKS
SECTION A This section comprises 20 multiple choice questions (MCQs) of 1 mark each.			
1.	Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined as $f(x) = x^3$, then f is : (A) one-one onto function (B) many-one function (C) one-one but not onto function (D) neither one-one nor onto function		
Sol	(A) one-one onto function		1
2.	The number of all possible matrices of order 2×3 with each entry 1 or 2 is : (A) 16 (B) 81 (C) 64 (D) 32		
Sol	(C) 64		1
3.	If $A = \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{bmatrix}$, then $A + A' = I$, if α is equal to : (A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{2}$ (D) $\frac{3\pi}{2}$		
Sol	$(B)\ \frac{\pi}{6}$		1
4.	If the area of a triangle is 15 sq. units with vertices $(3, -7)$, $(6, 3)$ and $(k, 3)$, then the value(s) of k is/are : (A) 3 (B) 9 (C) $-9, -3$ (D) 9, 3		
Sol	(D) 9, 3		1

5.	The interval in which $y = x^3.e^{-3x}$ is an increasing function is: $(A)\ (-1,2)$ (B) $(-\infty,2)$ $(C)\ (1,\infty)$ $(D)\ (0,1)$		
Sol	$(D)\ (0,1)$		1
6.	The value of k for which $f(x) = \begin{cases} \frac{\sin 7x}{5x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at x = 0 is : (A) $\frac{5}{7}$ (B) $\frac{7}{5}$ (C) 0 (D) $\frac{1}{5}$		
Sol	$(B)\ \frac{7}{5}$		1
7.	If $y = \tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$, then $\frac{dy}{dx}$ is equal to : (A) 1 (B) 2 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$		
Sol.	$(C)\ \frac{1}{2}$		1
8.	$\int \frac{\cos 2x}{\sin^2 x \cos^2 x}\, dx$ is equal to : (A) $\cot x + \tan x + C$ (B) $-\cot x + \sec x + C$ (C) $-\cot x + \tan x + C$ (D) $-(\tan x + \cot x) + C$		
Sol.	$(D)\ -(\tan x + \cot x) + c$		1
9.	$\int \frac{dx}{x^2 + 6x + 10}$ is equal to : (A) $x \tan^{-1}(x + 3) + C$ (B) $\tan^{-1}(x + 3) + C$ (C) $\frac{1}{3}\tan^{-1}(x + 3) + C$ (D) $\tan^{-1} x + C$		
Sol.	$(B)\ \tan^{-1}(x+3) + c$		1
10.	The general solution of the differential equation $\frac{dy}{dx} = e^{y-x}$ is : (A) $e^x - e^y = C$ (B) $e^{-y} + e^x = C$ (C) $e^{-y} - e^{-x} = C$ (D) $e^{-x} + e^{-y} = C$		

Sol.	$(C)\ e^{-y} - e^{-x} = C$		1
11.	The integrating factor of $(1 + y^2)\,dx = (\tan^{-1} y - x)\,dy$ is : (A) $\tan y$ (B) $\tan^{-1} y$ (C) $e^{\tan^{-1} y}$ (D) $e^{\tan^{-1} x}$		
Sol.	$(C)\ e^{\tan^{-1} y}$		1
12.	Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them. Then $\vec{a} + 2\vec{b}$ is a unit vector if : (A) $\theta = \frac{\pi}{2}$ (B) $\theta = \frac{2\pi}{3}$ (C) $\theta = \frac{3\pi}{4}$ (D) $\theta = \pi$		
Sol.	$(D)\ \theta = \pi$		1
13.	The area (in square units) of a parallelogram, whose diagonals are $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} - 3\hat{j} + 4\hat{k}$ is : (A) $2\sqrt{14}$ (B) $\sqrt{14}$ (C) $\sqrt{38}$ (D) $2\sqrt{6}$		
Sol.	$(B)\ \sqrt{14}$		1
14.	The corner points of the feasible region for an LPP are (0, 2), (6, 0), (6, 8) and (0, 5). Let Z = 4x + 6y be the objective function. (Maximum of Z) – (Minimum of Z) is equal to : (A) 60 (B) 48 (C) 42 (D) 18		
Sol.	(A) 60		1
15.	In an LPP, if the objective function has the same minimum value on two corner points of the feasible region, then the number of points at which the minimum value of objective function occurs is : (A) 0 (B) 2 (C) Infinite (D) Finite		
Sol.	(C) Infinite		1

16.	If events A and B are such that $P(A \mid B) = P(B \mid A)$, then : (A) $A \subset B$ but $A \neq B$ (B) $A = B$ (C) $A \cap B = \phi$ (D) $P(A) = P(B)$		
Sol	$(D)\ P(A) = P(B)$		1
17.	The probability of getting an odd prime number on each die, when a pair of dice is rolled, is : (A) $\frac{1}{18}$ (B) $\frac{1}{9}$ (C) $\frac{1}{36}$ (D) $\frac{1}{6}$		
Sol	$(B)\ \frac{1}{9}$		1
18.	In a family with three children, the probability that the eldest child is a boy, given that the family has at least one boy, is : (A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{4}{7}$		
Sol	$(D)\ \frac{4}{7}$		1

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled as Assertion (A) and the other labelled as Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.

(A) **Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).**

(B) **Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).**

(C) **Assertion (A) is true, but Reason (R) is false.**

(D) **Assertion (A) is false, but Reason (R) is true.**

19.	*Assertion (A) :* $f(x) = e^{-x}$ is a decreasing function, where $x \in \mathbb{R}$. *Reason (R) :* If $f'(x) < 0$, then $f(x)$ is a decreasing function.		

Sol	(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).		1
20.	*Assertion (A) :* For two independent events A and B, $P(A \mid B) = P(A)$. *Reason (R) :* A and B are independent events, then $P(A \cap B) = P(A) . P(B)$.		
Sol.	(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).		1
SECTION B ***This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.***			
21.	(a) For the matrix $A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$, find the numbers a and b so that $A^2 + aA + bI = O$. **OR** (b) Find the equation of line joining the points $(-1, -1)$ and $(1, 3)$, using determinants.		
Sol	$A^2 = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 4 \\ 8 & 3 \end{bmatrix}$	**I**	½
	Now, $A^2 + aA + bI = \begin{bmatrix} 11 & 4 \\ 8 & 3 \end{bmatrix} + \begin{bmatrix} 3a & a \\ 2a & a \end{bmatrix} + \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix} = O$	**II**	½
	$\Rightarrow \begin{bmatrix} 11+3a+b & 4+a \\ 8+2a & 3+a+b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	**III**	½
	Thus, $a = -4, b = 1$	**IV**	½
	OR		
	Let (x, y) be a point on the given line, so equation of line is $\begin{vmatrix} x & y & 1 \\ -1 & -1 & 1 \\ 1 & 3 & 1 \end{vmatrix} = 0$	**I**	1
	Thus, $-2x + y - 1 = 0$ required equation of line.	**II**	1

22.	(a) The volume of a spherical balloon is increasing at the rate of 20 cm^3/s. Find the rate of change of its surface area when its radius is 4 cm. **OR** (b) Find the intervals on which the function $f(x) = x^3 + 2x^2 - 1$ is strictly increasing.		
Sol 22(a)	Let V, S & r be the volume, surface area & radius of the spherical balloon respectively, so $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 20$ $\Rightarrow \frac{dr}{dt} = \frac{5}{\pi r^2}$ Now, $S = 4\pi r^2 \Rightarrow \frac{dS}{dt} = 8\pi r \frac{dr}{dt} = \frac{40}{r} = 10 \text{ cm}^2 / \text{sec}$	**I** **II** **III**	1 ½ ½
	OR		
22(b)	$f(x) = x^3 + 2x^2 - 1 \Rightarrow f'(x) = 3x^2 + 4x$ Put $f'(x) = 3x^2 + 4x = 0 \Rightarrow x = \frac{-4}{3}, x = 0$ Thus, function f is increasing on $\begin{cases} \left(-\infty, \frac{-4}{3}\right) \cup (0, \infty) \\ \text{or} \\ \left(-\infty, \frac{-4}{3}\right] \cup [0, \infty) \end{cases}$	**I** **II** **III**	1 ½ ½
23.	Find : $\int \frac{(x-4)\, e^x}{(x-2)^3}\, dx$		
Sol	$I \text{ (Say)} = \int \frac{x-4}{(x-2)^3} e^x dx = \int \left(\frac{1}{(x-2)^2} - \frac{2}{(x-2)^3} \right) e^x dx$ Let $f(x) = \frac{1}{(x-2)^2} \Rightarrow f'(x) = -\frac{2}{(x-2)^3}$ Thus, $I = e^x \frac{1}{(x-2)^2} + c$	**I** **II** **III**	1 ½ ½

24.	Find a unit vector perpendicular to both of the vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$, where A, B and C are (3, –1, 2), (1, –1, – 3) and (4, – 3, 1), respectively		
Sol	$\overrightarrow{AB} = -2\hat{i} + 0\hat{j} - 5\hat{k}, \overrightarrow{BC} = 3\hat{i} - 2\hat{j} + 4\hat{k}$ Now, $\overrightarrow{AB} \times \overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & -5 \\ 3 & -2 & 4 \end{vmatrix} = -10\hat{i} - 7\hat{j} + 4\hat{k}$ Thus, required unit vector $= \begin{cases} -\frac{10}{\sqrt{165}}\hat{i} - \frac{7}{\sqrt{165}}\hat{j} + \frac{4}{\sqrt{165}}\hat{k} \\ OR \\ \frac{10}{\sqrt{165}}\hat{i} + \frac{7}{\sqrt{165}}\hat{j} - \frac{4}{\sqrt{165}}\hat{k} \end{cases}$	**I** **II** **III**	½ 1 ½
25.	Find the image of the point (2, –1, 5) in the line $\vec{r} = (11\hat{i} - 2\hat{j} - 8\hat{k}) + \lambda(10\hat{i} - 4\hat{j} - 11\hat{k})$.		
Sol	Let $Q(10\lambda + 11, -4\lambda - 2, -11\lambda - 8)$ be the foot of the perpendicular drawn from P (2,-1,5) on the given line Now, direction ratio's of PQ are $<10\lambda + 9, -4\lambda - 1, -11\lambda - 13>$ direction ratio's of the given line are $<10, -4, -11>$ As PQ $\perp$ line, so $10(10\lambda + 9) - 4(-4\lambda - 1) - 11(-11\lambda - 13) = 0$ $\Rightarrow 100\lambda + 16\lambda + 121\lambda + 90 + 4 + 143 = 0$ $\Rightarrow \lambda = -1$ Thus, $Q(1, 2, 3)$ is the foot of the $\perp$ from P to the line Let the image be $R(a, b, c)$, so $a + 2 = 2,\ b - 1 = 4,\ c + 5 = 6 \Rightarrow R(0,\ 5\ ,1)$	**I** **II** **III** **IV**	½ ½ ½ ½

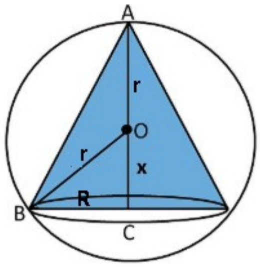
SECTION C ***This section comprises 6 Short Answer (SA) type questions of 3 marks each.***			
26.	(a) Find the value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)+\tan^{-1}\left(\tan\frac{5\pi}{6}\right)+\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$. **OR** (b) Prove that the function $f : N \to N$, $f(n) = (n^2 + n + 1)$ is one-one but not onto.		
26. (a) Sol.	$\sin^{-1}\left(\sin\frac{2\pi}{3}\right)+\tan^{-1}\left(\tan\frac{5\pi}{6}\right)+\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$ $=\sin^{-1}\left(\sin\frac{\pi}{3}\right)+\tan^{-1}\left(-\tan\frac{\pi}{6}\right)+\cos^{-1}\left(\cos\frac{\pi}{6}\right)$ $=\frac{\pi}{3}-\frac{\pi}{6}+\frac{\pi}{6}=\frac{\pi}{3}$	**I** **II**	1½ 1½
	OR		
26.(b)	Let $f(a)=f(b)$ for some $a,b\in\mathbb{N}$ $\Rightarrow a^2+a+1=b^2+b+1$ $\Rightarrow a^2-b^2=-(a-b)$ $\Rightarrow (a-b)(a+b+1)=0$ $\Rightarrow a=b$ as $a+b+1\neq 0$ $\therefore f$ is one-one function onto: Let $y=f(x)=x^2+x+1$, for some $y\in N$ For $y=1\in N, x^2+x=0$, which is not possible as $x\in\mathrm{N}$ Thus, $y=1$ has no pre-image $\therefore f$ is not onto function.	**I** **II** III IV	1 $\frac{1}{2}$ 1 ½
27.	(a) Find : $\int \frac{x}{\sqrt{7-6x-x^2}}\,dx$ **OR** (b) Find : $\int \frac{2}{(1-x)(1+x^2)}\,dx$		

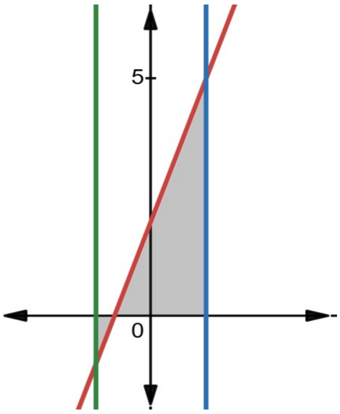
(a) Sol.	Let $I=\int\frac{x}{\sqrt{7-6x-x^2}}dx$ $I=\frac{-1}{2}\int\frac{-2x-6}{\sqrt{7-6x-x^2}}dx-3\int\frac{1}{\sqrt{7-6x-x^2}}dx$ $I=\frac{-1}{2}\left(2\sqrt{7-6x-x^2}\right)-3\int\frac{1}{\sqrt{4^2-(x+3)^2}}dx$ $I=-\sqrt{7-6x-x^2}-3\sin^{-1}\left(\frac{x+3}{4}\right)+c$	**I** **II** **III**	1 1 1
	OR		
27(b) Sol	Let $I=\int\frac{2}{(1-x)(1+x^2)}dx$ Let $\frac{2}{(1-x)(1+x^2)}=\frac{A}{(1-x)}+\frac{Bx+C}{(1+x^2)}$ For getting $A=B=C=1$ so, $I=\int\frac{1}{(1-x)}dx+\frac{1}{2}\int\frac{2x}{1+x^2}dx+\int\frac{1}{1+x^2}dx$ $I=-\log\lvert 1-x\rvert+\frac{1}{2}\log(1+x^2)+\tan^{-1}x+c$	**I** **II** **III**	½ 1 1½
28.	(a) Solve the differential equation : $x\frac{dy}{dx}=y-x\tan\frac{y}{x}$ **OR** (b) Solve the differential equation : $\frac{dy}{dx}=e^{x+y}+e^{x-y}$		
Sol. 28(a)	Given differential equation can be written as: $\frac{dy}{dx}=\frac{y}{x}-\tan\left(\frac{y}{x}\right)$ Put $y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$ so, $v+x\frac{dv}{dx}=v-\tan v\Rightarrow\cot v\,dv=\frac{-1}{x}dx$ $\Rightarrow\log(\sin v)=\log\left(\frac{c}{x}\right)\Rightarrow x\sin\left(\frac{y}{x}\right)=c$	**I** **II** **III** **IV**	½ 1 1 ½

	OR		
28(b) Sol.	As, $\frac{dy}{dx} = e^x\left(e^y + e^{-y}\right)$	I	½
	$\Rightarrow \int \frac{e^y}{e^{2y}+1}dy = \int e^x dx$	II	1
	$\therefore \tan^{-1}\left(e^y\right) = e^x + c$	III	1½
29.	Evaluate : $\int_0^{\pi} \frac{x\,dx}{1+\sin x}$		
Sol.	Let, $I = \int_0^{\pi} \frac{x\ dx}{1+\sin x} = \int_0^{\pi} \frac{(\pi - x)\ dx}{1+\sin x}$	I	½
	$2I = \pi\int_0^{\pi} \frac{1}{1+\sin x}dx = \pi\int_0^{\pi} \frac{1-\sin x}{\cos^2 x}dx$	II	1
	$2I = \pi\int_0^{\pi} \left(\sec^2 x - \sec x \tan x\right)dx = \pi\left(\tan x - \sec x\right)_0^{\pi}$	III	1
	$\therefore I = \pi$	IV	½
30.	If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}$, $\vec{b}$ and $\vec{c}$.		
Sol.	Given, $\left\|\vec{a}\right\| = \left\|\vec{b}\right\| = \left\|\vec{c}\right\|$ and $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$	I	1
	Let α, β, γ be the angle which $\left(\vec{a}+\vec{b}+\vec{c}\right)$ makes with $\vec{a}, \vec{b}$ and $\vec{c}$ repectively.		
	Now, $\cos\alpha = \frac{\left(\vec{a}+\vec{b}+\vec{c}\right).\vec{a}}{\left\|\vec{a}+\vec{b}+\vec{c}\right\|\left\|\vec{a}\right\|} = \frac{\left\|\vec{a}\right\|}{\left\|\vec{a}+\vec{b}+\vec{c}\right\|}$	II	1

	Similarly, $\cos\beta = \dfrac{\left\lvert\vec{b}\right\rvert}{\left\lvert\vec{a}+\vec{b}+\vec{c}\right\rvert}$ and $\cos\gamma = \dfrac{\left\lvert\vec{c}\right\rvert}{\left\lvert\vec{a}+\vec{b}+\vec{c}\right\rvert}$ Since, $\left\lvert\vec{a}\right\rvert = \left\lvert\vec{b}\right\rvert = \left\lvert\vec{c}\right\rvert \Rightarrow \cos\alpha = \cos\beta = \cos\gamma$ Thus, $\left(\vec{a}+\vec{b}+\vec{c}\right)$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$.	**III** **IV**	½ ½
31.	The corner points of the feasible region determined by the system of linear constraints in an LPP are (0, 10), (5, 5), (15, 15) and (0, 20). Let Z = px + qy, p, q > 0, be the objective function, then find the relation between p and q so that the maximum of Z occurs at both the points (15, 15) and (0, 20).		
Sol.	As, $Z = px + qy$ and given $Z_{\text{at }(15,15)} = Z_{\text{at }(0,20)}$ $\Rightarrow 15p + 15q = 0 + 20q$ Thus, $3p = q$	**I** **II** **III**	½ 2 ½
SECTION D ***This section comprises 4 Long Answer (LA) type questions of 5 marks each.***			
32. (a)	If A = $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations : $x - y + z = 4$ $2x + y - 3z = 0$ $x + y + z = 2$		
Sol 32(a)	For given matrix A, $\lvert A\rvert = 10 \neq 0$ (A^{-1} exists) so, adj A = $\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$ $\Rightarrow A^{-1} = \dfrac{1}{\lvert A\rvert}(\text{adj}A) = \dfrac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$ Now, given equations can be written in matrix form as	**I** **II** **III**	1 2 ½

	$\underset{A}{\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}}\underset{X}{\begin{bmatrix}x\\ y\\ z\end{bmatrix}}=\underset{B}{\begin{bmatrix}4\\ 0\\ 2\end{bmatrix}}\Rightarrow X=A^{-1}B$ $\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{10}\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & 5\\ 1 & -2 & 3\end{bmatrix}\begin{bmatrix}4\\ 0\\ 2\end{bmatrix}=\begin{bmatrix}2\\ -1\\ 1\end{bmatrix}$ $x=2, y=-1, z=1$	**IV** **V**	½ 1
	OR		
32. (b)	If $A=\begin{bmatrix}3 & 2\\ 7 & 5\end{bmatrix}$, $B=\begin{bmatrix}6 & 7\\ 8 & 9\end{bmatrix}$, then show that $(AB)^{-1}=B^{-1}A^{-1}$.		
32(b)Sol.	$AB=\begin{bmatrix}34 & 39\\ 82 & 94\end{bmatrix}\Rightarrow (AB)^{-1}=\frac{1}{-2}\begin{bmatrix}94 & -39\\ -82 & 34\end{bmatrix}$ Now, $A^{-1}=\begin{bmatrix}5 & -2\\ -7 & 3\end{bmatrix}$ & $B^{-1}=\frac{1}{-2}\begin{bmatrix}9 & -7\\ -8 & 6\end{bmatrix}$ Thus, $B^{-1}A^{-1}=\frac{1}{-2}\begin{bmatrix}9 & -7\\ -8 & 6\end{bmatrix}\begin{bmatrix}5 & -2\\ -7 & 3\end{bmatrix}=\frac{1}{-2}\begin{bmatrix}94 & -39\\ -82 & 34\end{bmatrix}$ $\therefore (AB)^{-1}=B^{-1}A^{-1}$	**I** **II** **III**	2 2 1
33. (a) **(b)**	Of all the closed right circular cylindrical cans of given volume 200 cm^3, find the dimensions of the can which has the minimum surface area. **OR** Show that the height of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$.		
33(a)Sol.	Let V, S, r and h be the volume, surface area, radius and height of the cylinder respectively. Given, $V=\pi r^2 h=200\Rightarrow h=\frac{200}{\pi r^2}$	**I** **II**	½ 1

	Thus, $S = 2\pi rh + 2\pi r^2 = 2\pi\left(\frac{200}{\pi r} + r^2\right)$ so, $\frac{dS}{dr} = 4\pi\left(\frac{-100}{\pi r^2} + r\right) = 0 \Rightarrow r = \left(\frac{100}{\pi}\right)^{\frac{1}{3}}$ $\Rightarrow \frac{d^2S}{dr^2} = 4\pi\left(\frac{200}{\pi r^3} + 1\right) = 12\pi > 0$ when $r = \left(\frac{100}{\pi}\right)^{\frac{1}{3}}$. $\therefore$ Surface area is minimum when $r = \left(\frac{100}{\pi}\right)^{\frac{1}{3}}$ cm and h $= 2\left(\frac{100}{\pi}\right)^{\frac{1}{3}}$ cm	**III** **IV** **V** **VI**	1 1½ ½ ½
	OR		
33(b)Sol.	Let V and R be the volume and radius of the cone respectively. Let us assume OC = x so, Height of the cone, $H = (r + x)$ and $x^2 + R^2 = r^2$  $V = \frac{1}{3}\pi R^2 H = \frac{1}{3}\pi\left(r^2 - x^2\right)\left(r + x\right)$ $V = \frac{1}{3}\pi\left(r + x\right)^2\left(r - x\right)$ $\Rightarrow \frac{dV}{dx} = \frac{1}{3}\pi\left(r + x\right)\left(r - 3x\right) = 0 \Rightarrow x = \frac{r}{3} \quad \left(\because r + x \neq 0\right)$ Now, $\frac{d^2V}{dx^2} = \frac{1}{3}\pi\left[-2r - 6x\right]$ $\left.\frac{d^2V}{dx^2}\right]_{x=r/3} = -2\pi r < 0$ $\therefore$ Volume is maximum at $x = \frac{r}{3}$. Now $h = r + \frac{r}{3} = \frac{4r}{3}$, hence shown.	**I** **II** **III** **IV** **V**	1 1 1 1 1
34.	Find the area of the region bounded by the line y = 3x + 2, the x-axis and the ordinates x = – 1 and x = 1.		

Sol	$A=-\int_{-1}^{\frac{-2}{3}}(3x+2)\,dx+\int_{\frac{-2}{3}}^{1}(3x+2)\,dx$ 	I	2
	$A=-\left(\frac{(3x+2)^2}{6}\right)_{-1}^{\frac{-2}{3}}+\left(\frac{(3x+2)^2}{6}\right)_{\frac{-2}{3}}^{1}$	II	2
	$\therefore$ Required Area $=\frac{1}{6}+\frac{25}{6}=\frac{13}{3}$	III	1
35.	Find the shortest distance between the lines : $\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(\hat{i}-3\hat{j}+2\hat{k})$, and $\vec{r}=(4\hat{i}+5\hat{j}+6\hat{k})+\mu(2\hat{i}+3\hat{j}+\hat{k})$.		
Sol	Here, $\overrightarrow{a_1}=\hat{i}+2\hat{j}+3\hat{k},\overrightarrow{a_2}=4\hat{i}+5\hat{j}+6\hat{k}$ $\overrightarrow{b_1}=\hat{i}-3\hat{j}+2\hat{k},\overrightarrow{b_2}=2\hat{i}+3\hat{j}+\hat{k}$	I	1
	so, $\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right)=3\hat{i}+3\hat{j}+3\hat{k}$	II	½
	$\overrightarrow{b_1}\times\overrightarrow{b_2}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&-3&2\\2&3&1\end{vmatrix}=-9\hat{i}+3\hat{j}+9\hat{k}$	III	1
	Shortest Distance $=\frac{\left\|\left(3\hat{i}+3\hat{j}+3\hat{k}\right).\left(-9\hat{i}+3\hat{j}+9\hat{k}\right)\right\|}{\sqrt{171}}$	IV	2
	Shortest Distance $=\frac{9}{\sqrt{171}}$ or $\frac{3}{\sqrt{19}}$	V	½

SECTION E

This section comprises 3 case-study based questions of 4 marks each.

	Case Study – 1		
36.	Two friends Ramesh and Divya have started playing Ludo at their house. While rolling the die, they observed each time, that the possible outcomes are {1, 2, 3, 4, 5, 6}. Let A be the set of players and B be the set of all possible outcomes, so A = {Ramesh, Divya} and B = {1, 2, 3, 4, 5, 6}. Based on the above information, answer the following questions : Let relation $R : B \to B$ be defined as $R = \{(x, y) : y \text{ is divisible by } x\}$. (i) Write all elements of R. *1* (ii) Find whether R is reflexive or not. *1* (iii) (a) Check whether R in B is symmetric or transitive. *2* **OR** (iii) (b) Let $R_1 : B \to B$ is defined as $R_1 = \{(x, y) : x = 2y, x, y \in B\}$. Check whether R_1 is reflexive, symmetric or transitive. *2*		
Sol 36.	(i){ (1,1), (2,2), (3,3), (4,4), (5,5) ,(1,2),(1,3),(1,4),(1,5),(2,4),(2,6), (3,6),(6,6)} (ii) R is reflexive as $\forall\, a \in B$, $(a, a) \in R$ (iii)(a) R is not symmetric but is transitive as $(1,2) \in R$ but $(2,1) \notin R$ OR (iii)(b)$R_1 = \{(2,1),(4,2),(6,3)\}$ R_1 is neither reflexive , nor symmetric nor transitive	**I** **I** **I** **I**	1 1 2 2
	Case Study – 2		

37.	A Math teacher of a school is teaching the derivative of function in parametric form, to her students. She explains that sometimes the relation between two variables, (say x and y), is neither explicit nor implicit, but the two variables are expressed in a third variable, (say t). So we have x = f(t) and y = g(t). Such form is called parametric form. For finding the derivative in such case we use the chain rule as $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} \Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}.$ Based on the above information, answer the following questions : (i) If $x = at^3$, $y = at^5$, then find $\frac{dy}{dx}$. *1* (ii) If $x = t + \frac{1}{t}$ and $y = t - \frac{1}{t}$, then find $\frac{dy}{dx}$. *1* (iii) (a) If x = 2 sin t + sin 2t and y = 2 cos t + cos 2t, then find $\frac{dy}{dx}$ at $t = \frac{\pi}{6}$. *2* **OR** (iii) (b) If $x = a\left(\cos t + \log \tan \frac{t}{2}\right)$, y = a sin t, then find $\frac{dy}{dx}$. *2*		
Sol	(i) $\frac{dx}{dt} = 3at^2, \frac{dy}{dt} = 5at^4 => \frac{dy}{dx} = \frac{5}{3}t^2$	**I**	1
	(ii) $\frac{dx}{dt} = 1 - \frac{1}{t^2}, \frac{dy}{dt} = 1 + \frac{1}{t^2} => \frac{dy}{dx} = \frac{t^2+1}{t^2-1}$	**I**	1
	(iii)(a) $\frac{dx}{dt} = 2cost + 2\cos 2t, \frac{dy}{dt} = -2sint - 2\sin 2t => \frac{dy}{dx} = -\frac{\sin 2t + sint}{cost + \cos 2t}$	**I**	1
$\left.\frac{dy}{dx}\right]_{t=\frac{\pi}{6}} =$	$\frac{\sin\frac{\pi}{3} + \sin\frac{\pi}{6}}{\cos\frac{\pi}{6} + \cos\frac{\pi}{3}} => \frac{dy}{dx} = -\frac{\frac{\sqrt{3}}{2} + \frac{1}{2}}{\frac{\sqrt{3}}{2} + \frac{1}{2}} = -1$	**II**	1
	OR		
	(iii)(b) $\frac{dx}{dt} = a\left(-sint + \frac{sec^2\frac{t}{2}}{\tan\frac{t}{2}} \cdot \frac{1}{2}\right) = a\left(-sint + \frac{1}{sint}\right) = a\frac{\cos^2 t}{sint}$	**I**	1
	$\frac{dy}{dt} = acost$; $\frac{dy}{dx} = \frac{sint}{cost} = tant$	**II**	1

	Case Study – 3		
38.	Two bags contain balls such that Bag I contains 5 red and 6 black balls, while Bag II contains 3 red and 4 black balls. Based on the above information, answer the following questions : (i) One ball is drawn at random from each of the two bags. Find the probability of getting both red balls. *1* (ii) One ball is drawn at random from each of the two bags. Find the probability of getting one red and one black ball. *1* (iii) (a) One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. Find the probability of getting a red ball. *2* **OR** (iii) (b) One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. Find the probability of getting a black ball. *2*		
Sol.	(i)$P(RR)=\frac{5}{11}\cdot\frac{3}{7}=\frac{15}{77}$	**I**	1
	(ii)P(One red and one black ball)= P(RB or BR)=$\frac{5}{11}\cdot\frac{4}{7}+\frac{6}{11}\cdot\frac{3}{7}=\frac{38}{77}$	**I**	1
	(iii)(a) P(Red from II bag)=P(RR)+P(BR) $=\frac{5}{11}\cdot\frac{4}{8}+\frac{6}{11}\cdot\frac{3}{8}=\frac{38}{88}=\frac{19}{44}$	**I**	2
	OR		
	(iii)(b) P(Black from II bag)=P(RB)+P(BB)=$\frac{5}{11}\cdot\frac{4}{8}+\frac{6}{11}\cdot\frac{5}{8}=\frac{50}{88}=\frac{25}{44}$	**I**	2